where a prime denotes transpose. Equation (4) is sometimes written in the form

$$(A' - \lambda_i B')y_i = 0 \tag{5}$$

and referred to as the *adjoint system* to Eq. (3). It is assumed that y_i and x_i are normalized with respect to B such that they satisfy the biorthogonality relations[8]

$$y_k' B x_i = \delta_{ik} \tag{6}$$

where δ_{ik} is the Kronecker delta.

The elements of A and B are assumed to be regular functions of certain parameters α_j $(j = 1,\ldots,m)$ in the neighborhood of a point α_j^0 $(j = 1,\ldots,m)$. The eigenvalues λ_i and eigenvectors x_i and y_i are therefore continuous functions of α_j in that neighborhood. Using the standard subscript notation for partial derivatives, our aim is to derive expressions for the following derivatives at the point α_j^0 $(j = 1,\ldots,m)$: $\lambda_{i,j}$; $\lambda_{i,jl}$; $x_{i,j}$; $y_{i,j}$.

First premultiply Eq. (3) by y_i' to obtain the scalar equation

$$y_i'(A - \lambda_i B)x_i = 0 \tag{7}$$

Differentiation of Eq. (7) with respect to α_j yields

$$y_{i,j}'(A - \lambda_i B)x_i + y_i'(A - \lambda_i B)x_{i,j} + y_i'(A_{,j} - \lambda_i B_{,j} - \lambda_{i,j} B)x_i = 0 \tag{8}$$

where, for instance, $A_{,j}$ denotes the matrix formed by differentiating the elements of A with respect to α_j. Using Eqs. (3, 4, and 6) leads immediately to the formula

$$\lambda_{i,j} = y_i'(A_{,j} - \lambda_i B_{,j})x_i \tag{9}$$

In order to obtain the second derivative of λ_i from Eq. (8) one needs information on the derivatives of x_i and y_i. It is convenient to write $x_{i,j}$ and $y_{i,j}$ in the following form:

$$x_{i,j} = \sum_{k=1}^{n} c_{ijk} x_k; \qquad y_{i,j} = \sum_{k=1}^{n} d_{ijk} y_k \tag{10}$$

Differentiation of

$$y_i' B x_i = 1 \tag{11}$$

with respect to α_j and then substitution of Eq. (10) into the result yields

$$\sum_{k=1}^{n} (c_{ijk} y_i' B x_k + d_{ijk} y_k' B x_i) + y_i' B_{,j} x_i = 0 \tag{12}$$

and hence, by application of Eq. (6),

$$c_{iji} + d_{iji} = -y_i' B_{,j} x_i \tag{13}$$

Also, by differentiating the relation

$$y_k'(A - \lambda_i B)x_i = 0 \tag{14}$$

and using Eqs. (3) and (10), one gets

$$y_k'(G_{ij} - \lambda_{i,j} B)x_i + \sum_{p=1}^{n} c_{ijp}(\lambda_p - \lambda_i) y_k' B x_p = 0 \tag{15}$$

where

$$G_{ij} = A_{,j} - \lambda_i B_{,j} \tag{16}$$

It follows that

$$c_{ijk} = (y_k' G_{ij} x_i)/(\lambda_i - \lambda_k) \quad \text{if} \quad k \neq i \tag{17}$$

Similar calculations using Eqs. (4) and (10) yield

$$d_{ijk} = (y_i' G_{ij} x_k)/(\lambda_i - \lambda_k) \quad \text{if} \quad k \neq i \tag{18}$$

Now consider the second derivative of λ_i with respect to α_j and α_l. Differentiating Eq. (8) with respect to α_l and using Eqs. (3, 4, 6, and 16) and equations similar to (8), one gets

$$\lambda_{i,jl} = y_i' G_{ij,l} x_i + y_i' G_{ij} x_{i,l} + y_{i,l}' G_{ij} x_i \tag{19}$$

The first term can be expressed in the form

$$y_i' G_{ij,l} x_i = y_i'(A_{,jl} - \lambda_i B_{,jl})x_i - (y_i' B_{,j} x_i)(y_i' G_{il} x_i) \tag{20}$$

with the use of Eqs. (9) and (16), while the second and third terms furnish

$$y_i' G_{ij} x_{i,l} + y_{i,l}' G_{ij} x_i = (c_{ili} + d_{ili}) y_i' G_{ij} x_i + \sum_{\substack{k=1 \\ k \neq i}}^{n} (c_{ilk} y_i' G_{ij} x_k + d_{ilk} y_k' G_{ij} x_i) \tag{21}$$

after substitution of Eq. (10). Application of Eqs. (13, 17, and 18) then leads to the following result:

$$\lambda_{i,jl} = y_i'(A_{,jl} - \lambda_i B_{,jl})x_i - (y_i' B_{,j} x_i)(y_i' G_{il} x_i) - (y_i' B_{,l} x_i)(y_i' G_{ij} x_i) + \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{[(y_i' G_{ij} x_k)(y_k' G_{il} x_i) + (y_i' G_{il} x_k)(y_k' G_{ij} x_i)]}{(\lambda_i - \lambda_k)} \tag{22}$$

where G_{ij} is defined in Eq. (16).

Some special cases are worth mentioning. The second derivative of λ_i with respect to a single α_j is given by

$$\lambda_{i,jj} = y_i'(A_{,jj} - \lambda_i B_{,jj})x_i - 2(y_i' B_{,j} x_i)(y_i' G_{ij} x_i) + 2 \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{(y_i' G_{ij} x_k)(y_k' G_{ij} x_i)}{(\lambda_i - \lambda_k)} \tag{23}$$

If the matrix B does not depend on α_j, then

$$\lambda_{i,j} = y_i' A_{,j} x_i \tag{24}$$

and

$$\lambda_{i,jj} = y_i' A_{,jj} x_i + 2 \sum_{\substack{k=1 \\ k \neq i}}^{n} \frac{(y_i' A_{,j} x_k)(y_k' A_{,j} x_i)}{(\lambda_i - \lambda_k)} \tag{25}$$

Expressions similar to Eqs. (24) and (25) were derived by means of a different procedure by Lancaster[5] for the special case in which $B = I$, the identity matrix, and A is a function of only one parameter. Lancaster also discussed the case where the eigenvalues λ_i are not all distinct. Finally, it is noted that all these results are applicable to self-adjoint systems, where $A' = A$ and $B' = B$, simply by setting $y_i = x_i$.

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Aerodynamic Characteristics of Vehicles Traveling in Perforated Tubes

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Nomenclature

A_G = cross-sectional area of guideway tube

A_H = area of perforations

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A_{LG} = lateral area of guideway tube
A_{OT} = cross-sectional area of outer tube
A_V = cross-sectional area of vehicle
A_{WG} = cross-sectional area of guideway tube wall
B = buoyancy force on vehicle
C_D = drag coefficient
$C_{M\delta}$ = moment coefficient due to angle of attack
d = distance from vehicle center of gravity to volumetric center
D = vehicle drag
D_V = vehicle diameter
L = vehicle length
OA = open area ratio
Re = vehicle Reynolds number
S_1 = lateral distance between perforations
S_2 = longitudinal distance between perforations
V = vehicle speed
α = vehicle blockage ratio
ρ = fluid density
ψ = tube ratio

Introduction

ONE of the promising high-speed ground transportation systems currently under investigation is that of a tube transportation system. In order to carry out proper evaluation of such a system it is necessary to obtain information about the aerodynamics of vehicles traveling within tubes. Among the many studies of tubed vehicle systems are the theoretical work of Goodman[1-3] concerning the static and dynamic stability derivatives for a body of revolution and the experimental work of Gouse[4,5] concerning drag coefficients for tubed systems. The aim of this Note is to provide an experimental verification of a portion of Goodman's results and to extend the drag results of Gouse to a wider range of parameters.

In particular, we consider a vehicle traveling within a perforated guideway tube that is surrounded by a larger concentric solid tube as shown in Fig. 1. Both the drag and the unstable aerodynamic moment on the vehicle are greatly affected by various parameters of the system. The results presented here show the effect of the following parameters: 1) vehicle blockage ratio, $\alpha = A_V/A_G$, 2) open area ratio, $OA = A_H/A_{LG}$ and 3) tube ratio, $\psi = A_G/(A_{OT} - A_{WG})$.‡

Experimental Apparatus

The data was obtained with a model system using a vertical, water filled plastic tube and gravity dropped vehicles that were circular in cross section with ogive noses and conical tails. Various vehicle blockage ratios were investigated but the vehicle length to diameter ratio was $L/D_V = 15$ for all cases.

An electromagnetic sensing system was used to measure the vehicle speed and orientation within the tube. This system consisted of a permanent magnet placed in the nose of the vehicle and two sets of coils mounted diametrically opposed on the outer tube wall (see Fig. 1). The signals produced by the vehicle passing the coils were amplified and recorded on a strip chart. The vehicle speed was easily determined from the frequency of the sinusoidal signals produced. Also, since the magnitude of the signal depended on the vehicle speed and distance from the tube wall, the relative magnitude of the signals provided a means of determining if the vehicle were stable (not oscillating) or unstable (oscillating).

A movable weight in the vehicle was adjusted until the vehicle was neutrally stable to oscillations in angle of attack. At this condition the destabilizing aerodynamic moment about the vehicle center of gravity is balanced by the stabilizing buoyancy moment. Hence the moment coefficient due to angle of attack, $C_{M\delta}$, can be written as

$$C_{M\delta} = Bd/\tfrac{1}{2}\rho V^2 A_V L$$

‡ It is noted that for very thin walled perforated tubes the tube ratio is essentially the square of the ratio of the perforated tube diameter to the solid outer tube diameter.

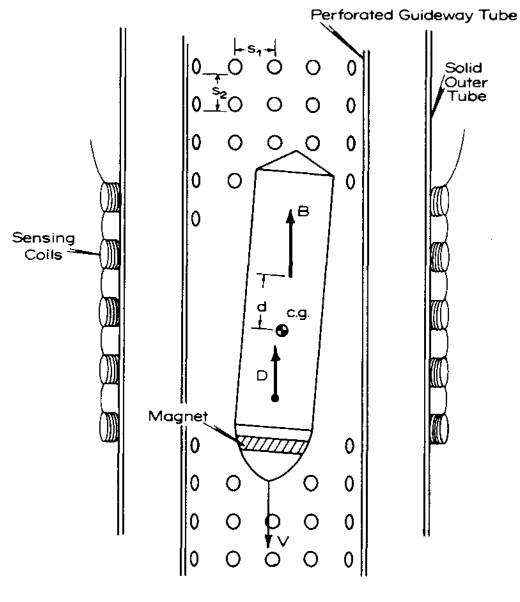

Fig. 1 Vehicle guideway system.

Likewise the drag coefficient is given by

$$C_D = D/\tfrac{1}{2}\rho V^2 A_V$$

The data for this study is for a Reynolds number (based on the vehicle speed and diameter) of $Re = 4 \times 10^4$.

The perforations of the inner guide tube consisted of circular holes placed in a rectangular array with $S_1/S_2 = 0.705$.

Results

The drag coefficient data presented in Fig. 2 for a vehicle blockage ratio of $\alpha = 0.64$ shows that the open area ratio, OA, and tube ratio, ψ, have a great effect on the vehicle drag. It is noted that $OA = 0.0\%$ corresponds to a solid inner tube while

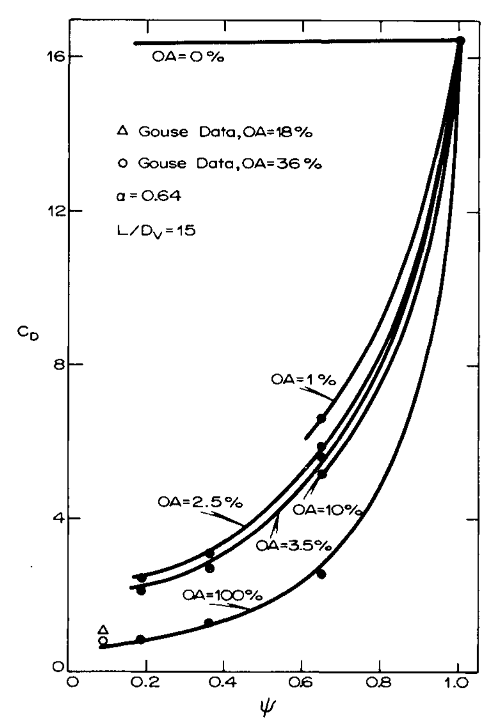

Fig. 2 Drag coefficient as a function of tube ratio and open area ratio.

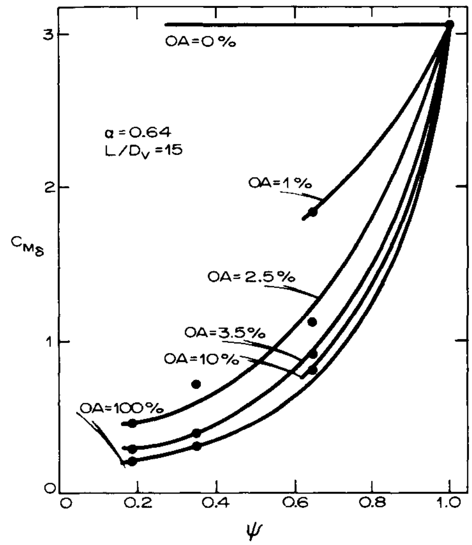

Fig. 3 Moment coefficient as a function of tube ratio and open area ratio.

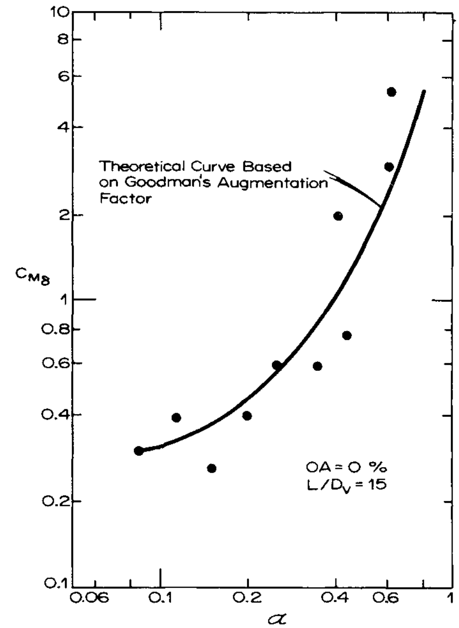

Fig. 4 Moment coefficient as a function of vehicle blockage ratio.

$OA = 100\%$ corresponds to an imaginary inner tube. Also, $\psi = 0$ corresponds to a perforated tube within an infinitely large outer tube (free air) while $\psi = 1$ corresponds to equal diameter porous tube and outer solid tube. It is seen that use of a slight amount of perforations and a guideway tube only slightly smaller than the solid outer tube can cause considerable drag reduction. Experimental results of Gouse are also shown on the same figure.

Moment coefficient data is shown in Fig. 3 for a vehicle blockage ratio of $\alpha = 0.64$. It is seen that the moment coefficient results are very similar to the drag results as far as dependence on open area ratio and tube ratio. Thus an increase in open area ratio or a decrease in tube ratio can greatly reduce the drag experienced by a tubed vehicle and also greatly reduce the unbalanced aerodynamic moment on the vehicle.

Confinement of a vehicle within a tube greatly alters the aerodynamic forces on the vehicle compared to the free air case. Both the drag and moment coefficients increase rapidly with blockage ratio. Gouse[4,5] presents considerable data to illustrate this for the drag forces. Figure 4 illustrates the dependence of the moment coefficient on the blockage ratio for a solid walled guide tube ($OA = 0.0\%$). As the vehicle diameter becomes more nearly the diameter of the guide tube the unbalanced aerodynamic moment increases rapidly.

Goodman[1] provides an augmentation factor for the moment coefficient (Fig. 2 of Ref. 1) that was obtained by considering incompressible inviscid slender-body theory applied to a body of revolution traveling in a tube. The augmentation factor, which is a function of vehicle blockage ratio, represents the ratio of the moment coefficient for the vehicle within the tube compared to its free air (no tube) value. To compare this theoretical result with the present experimental result the data of Fig. 4 was extrapolated to $\alpha = 0$ (free air case). This free air case ($C_{M\delta} = 0.25$ for $\alpha = 0$) multiplied by Goodman's augmentation factor is shown as the solid curve of Fig. 4.

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